The decoherence dynamics of the multipartite entanglement in non-Markovian environment

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Abstract. We consider four two-level atoms interacting with independent non-Markovian reservoirs with detuning. We mainly investigate the effects of the detuning and the length of the reservoir correlation time on the decoherence dynamics of the multipartite entanglement. We find that the time evolution of the entanglement of atomic and reservoir subsystems is determined by a parameter, which is a function of the detuning and the reservoir correlation time. We also find that the decay and revival of the entanglement of the atomic and reservoir subsystems are closely related to the sign of the decay rate. We also show that the cluster state is the most robust to decoherence comparing with Dicke, GHZ, and W states for this decoherence channel.

1. Introduction

Entanglement plays a central role in quantum information processing. Since the maintenance of entangled states is very important in quantum information processing systems, the study of the effect of decoherence on entangled states would be of considerable importance from theoretical as well as experimental point of view. Recently entanglement of open quantum systems has attracted considerable attention due to its significance for both fundamentals and applications of quantum information processing [1, 2, 3, 4, 5, 6, 7]. Yu and Eberly [3] found that the decay of a single qubit coherence can be slower than the decay of entanglement, and they named the abrupt disappearance of entanglement at a finite time entanglement sudden death (ESD), which has been experimentally tested very recently [8]. Later López *et al.* [5] revealed that when the bipartite entanglement suddenly disappears, the entanglement of corresponding reservoir suddenly and necessarily appears, which is called entanglement sudden birth (ESB). Although the bipartite entanglement is well understood in many aspects, until now the multipartite entanglement is far from clear and thus deserves profound understandings. Several important applications of multipartite entangled states, like quantum error correction, quantum computing, *etc.* have been found (for recent reviews see Refs.[9, 10, 11] and references therein). Recently, people investigated the decoherence dynamics of multipartite entangled states, for instance, Gühne, Bodoky and Blaauboer [12] discussed some multipartite entanglement properties under the influence of a global dephasing process using the geometric measure of entanglement, and they showed that the Dicke state is the most robust to decoherence comparing with GHZ, cluster, and W states. Borras and Majtey *et al.* [13] investigated the robustness of highly entangled multiqubit states under different decoherence channels [14], and later, they further studied the geometry of robust entangled multiparticle states under decoherence [15].

In realistic physical systems the assumption of Markovian dynamics can only be an approximation. Generally speaking the quantum system of interest interacts with the reservoirs, whose spectral density strongly varies with frequency, which is called non-Markovian open quantum systems [16]. Non-Markovian dynamics is characterized by the existence of a memory time scale during which information that has been transferred from the system to the environment can flow back into the system. The non-Markovian systems appear in many branches of physics, such as quantum optics [16, 17, 18], solid state physics [19], and quantum chemistry [20]. In quantum information processing, the non-Markovian character of decoherence has to be considered [21]. The non-Markovian dynamics has drawn great attentions including continuous-variable [22, 23, 24, 25, 26, 27, 28] and discrete-variable systems [29, 30, 31, 32, 33, 34, 35, 36, 37]. Very recently N qubits initially in the mixed GHZ-type and W-type states interacting with independent structured reservoirs have been investigated and it is found that the N-qubit entanglement revivals are related to the qubit number N and the purity of the initial state of N qubits [35]. The measure of the degree of non-Markovian behavior [38] and a basic relation between the quantum Fisher

information flow and non-Markovianity have been proposed for quantum dynamics of open systems [39].

In this paper, we are very interested in the decoherence dynamics of multipartite entanglement under the influence of non-Markovian reservoir. We mainly investigate the effects of the detuning and the length of the reservoir correlation time on the decoherence dynamics of the multipartite entanglement. We consider that four noninteracting two-level atoms interact with four independent non-Markovian reservoirs with detuning. We let the four atoms initially be prepared in different four particle entangled states (cluster, Dicke, GHZ, and W states). We find that the dynamical behaviors of the entanglement of the atomic and reservoir subsystems for various initial states are determined by a complex parameter, which is a function of the detuning and the reservoir correlation time. The real part of this parameter is closely related to the decay of the entanglement and the imaginary part of it is closely related the oscillations of the revivals, and all the dynamical behavior of the entanglement can be uniformly explained by this parameter. We also find a close relationship between the decay rate and the entanglement of the atomic and reservoir subsystems, i.e., the decay or revival of the entanglement of the atomic and reservoir subsystems are decided by the sign of the decay rate. We also find that the cluster state is the most robust to decoherence, which is different from the results of Ref.[12]. Here we consider the amplitude damping process, while Ref.[12] discuss the global dephasing process. The paper is organized as follows: In Sec. 2, we present the model which consists of four atoms interacting with individual non-Markovian reservoirs, and introduce the multipartite entanglement measure. We present and analyze the results in Sec. 3. Finally, we give some conclusions in Sec. 4.

2. Model

We consider a system of four identical noninteracting two-level atoms, each of them coupled to its own reservoir. For simplicity, we assume that each corresponding reservoir is initially in the vacuum state. Due to the independence of each atom, we only need to discuss the problem of a single atom interacting with its corresponding reservoir. The Hamiltonian of the interaction between a single atom and N-mode reservoir under the rotating-wave approximation can be written as ($\hbar = 1$),

$$\hat{H} = \omega_a \hat{\sigma}_+ \hat{\sigma}_- + \sum_{k=1}^{N} \omega_k \hat{a}_k^\dagger \hat{a}_k + \sum_{k=1}^{N} g_k (\hat{\sigma}_- \hat{a}_k^\dagger + \hat{\sigma}_+ \hat{a}_k), \tag{1}$$

where $\hat{\sigma}_+ = |1\rangle\langle 0|$ and $\hat{\sigma}_- = |0\rangle\langle 1|$, are the Pauli raising and lowering operators for the atom respectively, and ω_a is the transition frequency of each atom. $\hat{a}_k^\dagger$ and $\hat{a}_k$ are the creation and annihilation operators with frequency ω_k for the reservoir mode k, and g_k is the corresponding coupling constant.

Here we consider one excitation case, namely, the atom and corresponding reservoir are initially in the excited state and vacuum state respectively, i.e., $|\psi_0\rangle = |1\rangle_a \otimes |\bar{0}\rangle_r$, where $|\bar{0}\rangle_r = \prod_{k=1}^{N} |0_k\rangle_r$. The subscripts a and r refer to the atom and the corresponding

reservoir respectively. So the state of the total system at any time t can be denoted by

$$|\psi_t\rangle = \nu(t)|1\rangle_a|\bar{0}\rangle_r + \sum_{k=1}^{N} D_k(t)|0\rangle_a|1_k\rangle_r, \tag{2}$$

where the state $|1_k\rangle_r$ represents the reservoir having one excitation in mode k.

Similar to the method used in Ref.[16], we can obtain a closed equation for the coefficient $\nu(t)$ in Eq.(2),

$$\dot{\nu}(t) = -\int_0^t f(t-t_1)\nu(t_1)dt_1, \tag{3}$$

where the kernel $f(t-t_1) = \int d\omega J(\omega)\exp[i(\omega_a - \omega)(t-t_1)]$ is related to the spectral density $J(\omega)$ of the reservoir. We take a Lorentzian spectral density of the reservoir [16, 38]

$$J(\omega) = \frac{1}{2\pi}\frac{\gamma_0\lambda^2}{(\omega_a - \delta - \omega)^2 + \lambda^2}, \tag{4}$$

the center of which is detuned from the transition frequency ω_a of the two-level atom by an amount δ. And the parameter λ defines the spectral width of the coupling, which is associated with the reservoir correlation time by the relation $\tau_B = \lambda^{-1}$ and the parameter γ_0 is related to the relaxation time scale τ_R by the relation $\tau_R = \gamma_0^{-1}$.

By making the Laplace transformation of Eq.(3), we can obtain the solution of $\nu(t)$ with initial condition $\nu(0) = 1$,

$$\nu(t) = e^{-(\lambda - i\delta)t/2}[\cosh(\chi t/2) + \frac{\lambda - i\delta}{\chi}\sinh(\chi t/2)], \tag{5}$$

where

$$\chi = \sqrt{(\lambda - i\delta)^2 - 2\gamma_0\lambda}. \tag{6}$$

Furthermore, if we take the form of a collective state of the reservoir, namely, letting $|\bar{1}\rangle_r = (1/\mu(t))\sum_{k=1}^{N} D_k(t)|1\rangle_r$ with $\mu(t) = \sqrt{1 - |\nu(t)|^2}$, Eq.(2) can be rewritten as

$$|\psi_t\rangle = \nu(t)|1\rangle_a|\bar{0}\rangle_r + \mu(t)|0\rangle_a|1\rangle_r. \tag{7}$$

There are many entanglement measures to quantify the bipartite entanglement such as von Neumann entropy, negativity [40, 41], concurrence [42], relative entropy [43], *etc.* However, generally speaking, for multipartite system the definition of entanglement measure is difficult. Up to now, there are some entanglement measures proposed to quantify the multipartite entanglement, which include Schmidt measure [44], the geometric measure of entanglement [45], the global entanglement measure [46, 47], *etc.* In this paper, we will use a very popular entanglement measure [13, 15] which is averaged over all possible bipartitions. The mathematical definition of this measure is

$$E = \frac{1}{[N/2]}\sum_{m=1}^{[N/2]} E^{(m)}, \tag{8}$$

where

$$E^{(m)} = \frac{1}{N^m_{\text{bipart}}} \sum_{i=1}^{N^m_{\text{bipart}}} E(i). \tag{9}$$

$E(i)$ indicates the entanglement connected to single bipartition of the N-qubit system, and $E^{(m)}$ denotes the average entanglement over all nonequivalent bipartitions N^m_{bipart} between subsets of m qubits and the remaining $N-m$ qubits. If one uses the linear entropy S_L of the reduced density matrix of the smaller bipartitions to compute $E(i)$, it will reduce to the well known Meyer-Wallach multipartite entanglement measure [46]. Because in this paper we will deal with mixed states, we take the negativity to measure the bipartite entanglement. The normalized negativity is defined as [15]

$$E(i) = \frac{2}{2^m - 1} \sum_i |\alpha_i|, \tag{10}$$

where α_i is the negative eigenvalue of the partial transpose matrix for m and the remaining $N-m$ bipartition.

3. Results and discussions

We suppose that four two-level atoms are initially prepared in some famous multipartite entangled states (cluster, Dicke, GHZ, and W states), and the corresponding reservoirs are initially prepared in the vacuum state. For W state the initial state of atom-reservoir system is:

$$\begin{aligned} |\phi_0\rangle = (&|0001\rangle_{a_1a_2a_3a_4} + |0010\rangle_{a_1a_2a_3a_4} + |0100\rangle_{a_1a_2a_3a_4} \\ &+ |1000\rangle_{a_1a_2a_3a_4})|\bar{0}\bar{0}\bar{0}\bar{0}\rangle_{r_1r_2r_3r_4}/2, \end{aligned} \tag{11}$$

where the subscripts $a_i (i = 1, 2, 3, 4)$ represents the atom, and $r_i (i = 1, 2, 3, 4)$ refers to the corresponding reservoir. From Eq.(7), the evolution of the total system can be obtained

$$\begin{aligned} |\phi_t\rangle = [&|000\rangle_{a_1a_2a_3} |\bar{0}\bar{0}\bar{0}\rangle_{r_1r_2r_3} (\nu(t)|1\rangle_{a_4}|\bar{0}\rangle_{r_4} + \mu(t)|0\rangle_{a_4}|1\rangle_{r_4}) \\ &+ |000\rangle_{a_1a_2a_4} |\bar{0}\bar{0}\bar{0}\rangle_{r_1r_2r_4} (\nu(t)|1\rangle_{a_3}|\bar{0}\rangle_{r_3}) + \mu(t)|0\rangle_{a_3}|1\rangle_{r_3}) \\ &+ |000\rangle_{a_1a_3a_4} |\bar{0}\bar{0}\bar{0}\rangle_{r_1r_3r_4} (\nu(t)|1\rangle_{a_2}|\bar{0}\rangle_{r_2}) + \mu(t)|0\rangle_{a_2}|1\rangle_{r_2}) \\ &+ |000\rangle_{a_2a_3a_4} |\bar{0}\bar{0}\bar{0}\rangle_{r_2r_3r_4} (\nu(t)|1\rangle_{a_1}|\bar{0}\rangle_{r_1} + \mu(t)|0\rangle_{a_1}|1\rangle_{r_1})]/2. \end{aligned} \tag{12}$$

So the reduced density operator of atomic subsystem $\rho_a(t) = \mathrm{Tr}_r(|\phi_t\rangle\langle\phi_t|)$ and the reduced density operator of reservoir subsystem $\rho_r(t) = \mathrm{Tr}_a(|\phi_t\rangle\langle\phi_t|)$ can be obtained. Then from Eq.(8-10) we can obtain the degree of the entanglement of the atomic and the reservoir subsystems. For convenience, we denote the degree of the entanglement of the atomic subsystem and the corresponding reservoir subsystem by E_a and E_r, respectively. Here for W state, E_a and E_r can be obtained,

$$E_a = |[16 - 16|\nu(t)|^2 - (6\sqrt{7|\nu(t)|^4 - 8|\nu(t)|^2 + 4} + 4\sqrt{2|\nu(t)|^4 - 2|\nu(t)|^2 + 1})]|/24, \tag{13}$$

and

$$E_r = |[16|\nu(t)|^2 - (6\sqrt{7|\nu(t)|^4 - 6|\nu(t)|^2 + 3} + 4\sqrt{2|\nu(t)|^4 - 2|\nu(t)|^2 + 1})]|/24. \quad (14)$$

Similarly we can obtain E_a and E_r for the other initial states, cluster state $|CL_4\rangle = (|0000\rangle + |0011\rangle + |1100\rangle - |1111\rangle)/2$, Dicke state $|D_4\rangle = (|0011\rangle + |0101\rangle + |1001\rangle + |1100\rangle + |0110\rangle + |1010\rangle)/\sqrt{6}$, and GHZ state $|DHZ_4\rangle = (|0000\rangle + |1111\rangle)/\sqrt{2}$, respectively.

First we consider the resonant case, i.e., $\delta = 0$. (i) $\lambda = 10\gamma_0$, which corresponds to the Markovian regime. In Fig.1 we plot the entanglement evolution of the atomic and reservoir subsystems for initial cluster, Dicke, GHZ, and W states. From Fig.1a we can see that the cluster state is the most robust against decoherence. The result is quite different from that of Ref.[12], in which the Dicke state is the most robust against decoherence among GHZ, cluster, and W states. This is because that the decoherence channel used in Ref.[12] is different from ours. In Ref.[12] the global dephasing channel is considered, while in this paper we consider the amplitude damping channel. This means that the most robust multipartite entangled state might be different for different decoherence channels. It should be noted that the entanglement measure different from ours is used in Ref.[12], but we use our entanglement measure to recalculate the entanglement of Ref.[12], and the result is the same as that of Ref.[12]. From Fig.1 we can see that the entanglement of the atomic subsystem for all the initial states decreases monotonically to zero, the entanglement of the reservoir subsystem for all the initial states increases monotonically to the steady maximum, and the entanglement contained initially in the atomic subsystem is finally transferred into the reservoir subsystem.

(ii) $\lambda = 0.1\gamma_0$, which corresponds to the non-Markovian regime with relatively short reservoir correlation time. In Fig.2 we plot the entanglement evolution of the atomic and reservoir subsystems for initial cluster, Dicke, GHZ, and W states with $\lambda = 0.1\gamma_0$ and $\delta = 0$. The dynamical behaviors of the entanglement of the atomic and reservoir subsystems in the non-Markovian regime are quite different from that in the Markovian regime. It can be seen from Fig.1a and Fig.2a that the common feature for different initial states in the non-Markovian regime is that the entanglement of the atomic subsystem decreases to zero much slowly than that in the Markovian regime, and in the non-Markovian regime after the entanglement of the atomic subsystem decays to zero it can revive at later time, which is quite different from the Markovian case. The reason is that the information, which the atomic subsystem loses to the reservoir, is later recovered by the atomic subsystem due to the reservoir non-Markovian memory. It is noted that in both the Markovian and non-Markovian regimes all the initial entanglement of atomic subsystem E_a will decay and is eventually lost for long times, and the entanglement of reservoir subsystem E_r gradually increases to the steady maximum from zero, which can be seen from Figs.1 and 2. It can be seen from Figs.1b and 2b that in the non-Markovian regime the entanglement of the reservoir subsystem E_r at first shows oscillations as a function of time for all the initial atomic states, and finally the steady maximum entanglement is achieved, while in the Markovian regime E_r increase

to the steady maximum monotonically. All the distinction between the entanglement properties in the Markovian regime and that in the non-Markovian regime is induced by the non-Markovian memory. In other words, in the Markovian regime the information flow is one directional, namely from atoms to reservoirs, while in the non-Markovian regime the information flow is bidirectional, namely the exchange of information back and forth between the atomic and reservoir subsystems, which causes the oscillations of the entanglement of the atomic and reservoir subsystems.

(iii) $\lambda = 0.01\gamma_0$, which corresponds to the non-Markovian regime with the relatively long reservoir correlation time. In Fig.3 we plot the entanglement evolutions of atomic and reservoir subsystems for the four initial states with $\lambda = 0.01\gamma_0$ and $\delta = 0$. Comparing Figs.2a and 3a, we can find that the revival of the E_a with relatively long reservoir correlation time is more obvious than that with relatively short reservoir correlation time, i.e., the amplitude of revival with relatively long reservoir correlation time is much larger than that with relatively short reservoir correlation time. For reservoir subsystem, compared with the case with relatively short reservoir correlation time, it is more difficult to achieve the steady maximum of entanglement with relatively long reservoir correlation time, which can be seen from Figs.2b and 3b. This can be understood as follows: Increasing the reservoir correlation time means that the memory effect of the reservoir becomes stronger, and then the amount of information exchanged between the atomic and the reservoir subsystems will be enhanced. So the atomic subsystem can obtain more information from the reservoir subsystem in the case of relatively long reservoir correlation time and the revival is stronger, and because of the enhanced information exchange back and forth the reservoir subsystem will need more time to achieve the final maximum entanglement.

Now we consider the off-resonant case, i.e., $\delta = 8\lambda$. (i) $\lambda = 0.1\gamma_0$. In Fig.4 we plot the entanglement evolution of atomic and reservoir subsystems for the four initial states with $\delta = 8\lambda$ and $\lambda = 0.1\gamma_0$. In the off-resonant case the entanglement of the atomic subsystem E_a decays to zero with small oscillations, and during each oscillation E_a can not collapse to zero. And the overall decay rate is smaller than that in the corresponding resonant case, which can be seen from Figs.4a and 2a. The entanglement of the reservoir subsystem E_r at first increases with very small amplitude oscillations in a very short period of time and then increases monotonically to the steady entanglement, and the overall increasing rate is smaller than that in the corresponding resonant case, which can be seen from Figs.4b and 2b. This can be easily understood: When the value of the detuning δ increases, the effective coupling between the atomic and reservoir subsystem decreases. So the exchange of information between the atomic subsystem and the reservoir subsystem is not effective and adequate. (ii) $\lambda = 0.01\gamma_0$. In Fig.5 we plot the entanglement evolution of atomic and reservoir subsystems for the four initial states with $\delta = 8\lambda$ and $\lambda = 0.01\gamma_0$. Comparing Figs.5b and 3b we can find that due to the increasing of δ, the exchange of information is not effective, the oscillations of E_r are not adequate, more specifically E_r can not achieve its maximum during each oscillation. From Figs.5a and 3a it can be found that increasing the detuning δ the

period of the revival is shorten, and the amplitude of the revival increases. This result is very interesting. As we have mentioned above increasing the detuning δ will make the exchange of information less effective, then why the revival becomes stronger? Now we analyze the decoherence dynamics of the multipartite entangled states in detail.

To gain insight in the physical processes characterizing the decoherence dynamics for different initial states, we consider the parameter χ, and find that all the above phenomenon can be uniformly explained by this parameter. From Figs.1-5 it is easy to find that the dynamical behaviors of the entanglement for different initial states are very similar. For simplicity in the following we will take W state as an example. From Eq.(6) we can see that generally χ is a complex number, and we will show that the real part Reχ is responsible for the decay of E_a and the imaginary part Imχ is responsible for the oscillations associated with the revival. From Eq.(13) we can see that the degree of entanglement is a function of $|\nu(t)|^2$, which means that all the decoherence dynamics of the entanglement entirely depends on $\nu(t)$. And it is easy to see from Eq.(5) that in the long time limit $\nu(t)$ is dominated by the terms containing the factor $e^{(-\lambda+|\mathrm{Re}\chi|)t/2}$. From numerical calculations we find that $|\mathrm{Re}\chi|$ increases with δ and is always less than λ, and $|\mathrm{Im}\chi|$ also increases with δ. From Eqs.(5) and (13) roughly speaking, $\lambda - |\mathrm{Re}\chi|$ determines the decay of the entanglement, which we call it the decay exponent to be distinguished from the decay rate $\gamma(t)$ [16], and $|\mathrm{Im}\chi|$ determines the basic frequency of the oscillations in the revivals (it is noted that the overall phase factor $e^{i\delta t/2}$ in $\nu(t)$ does not make any contributions to the entanglement). When $\lambda > 2\gamma_0$ and $\delta = 0$, from Eq.(6) χ is a real number, i.e., $|\mathrm{Im}\chi| = 0$, which corresponds to the Markovian regime. Hence E_a will decay exponentially to zero without oscillations, and the revival can not appear. It is easy to prove that when $\lambda > 2\gamma_0$, the decay exponent $\lambda - \sqrt{\lambda^2 - 2\gamma_0\lambda}$ is a decreasing function of λ, and approaches γ_0 with the increasing λ, the maximum value of which is $2\gamma_0$ occurring at $\lambda = 2\gamma_0$. When $\delta = 0$ and $\lambda < 2\gamma_0$, which is corresponding to the non-Markovian regime, χ is a pure imaginary, and the oscillations appear. In this case the decay exponent is just λ. That is why the entanglement decay for $\lambda = 0.1\gamma_0$ corresponding to the non-Markovian regime (Fig.2a) is slower than that $\lambda = 10\gamma_0$ corresponding to the Markovian regime (Fig.1a). And also that is why the entanglement with $\lambda = 0.01\gamma_0$ and $\delta = 0$ (Fig.3a) decays much slowly than that with $\lambda = 0.1\gamma_0$ and $\delta = 0$ (Fig.2a). Remember that $|\mathrm{Re}\chi|$ and $|\mathrm{Im}\chi|$ increase with δ, so the decay exponent $\lambda - |\mathrm{Re}\chi|$ decreases with the increasing of δ. In this way the envelope of $E_a(t)$ decay more and more slowly with the increasing of δ, so that during each revival the amplitude achieved is increasing with the increasing of δ. This explains why with the increasing the detuning δ the period of the revival is shorten, and the amplitude of the revival increases (see Figs.3a and 5a). Now we consider the dispersive regime, i.e., $\delta \gg \lambda, \gamma_0$, and in this case in the long time limit $\nu(t) \approx 1 - i\lambda^2/4\delta^2$ and the steady entanglement of the corresponding atomic subsystem $E_a \approx [6\sqrt{3} + 4 + (20 + 6\sqrt{3})\lambda^4/16\delta^4]/24$ can be achieved. This means that in the dispersive regime the decay of entanglement E_a is strongly inhibited. We also calculate the degree of entanglement for the initial five (and six) particle W state, and we find that the entanglement dynamics of atomic subsystem

for the initial five (or six) particle W state is almost the same as that of the initial four particle W state, more specifically the influence of the detuning and the length of the reservoir correlation time on the dynamical behavior of the entanglement for the initial five (or six) particle W state is almost the same as that for the initial four particle W state. Whenever the degree of entanglement for the four particle case increases, the degree of entanglement for the corresponding five (or six) particle case also increases, and whenever the degree of entanglement for the four particle case decreases, the degree of entanglement for the five (or six) particle case also decreases. And the time, at which the entanglement reaches the maximum (or the minimum), is almost the same for all the three cases. This can be understood. Because we find that the entanglement of the atomic subsystem for five (or six) particle case is also a function of $|\nu(t)|^2$, which means that the dynamical behavior of the entanglement for the five (or six) particle is also decided by the real part and imaginary part of $|\chi|$.

It is well known that in the most general form of a time-local master equation for the reduced density operator, the decoherence is induced by the Lindblad (jump) operator with a decay rate $\gamma(t)$. If the decay rate $\gamma(t)$ is always positive, this describes the so-called time-dependent Markovian process [48, 29, 49], but if at least during a period of time the decay rate $\gamma(t)$ is negative, the non-Markovian process emerges. Now we also take W state as an example to show the relation between the decoherence dynamics of the entanglement and the decay rate $\gamma(t)$. For simplicity we let $\lambda = 0.01\gamma_0$ and $\delta = 0$, and in this case the decay rate $\gamma(t)$ can be expressed as $\gamma(t) = -2\mathrm{Re}\{\dot{\nu}(\mathrm{t})/\nu(\mathrm{t})\}$ [16], where $\nu(t)$ is obtained by choosing δ=0 in Eq.(5). In Fig.6 we plot E_a, E_r and $\gamma(t)$ as functions of scaled time $\gamma_0 t$ for $\lambda = 0.01\gamma_0$ and $\delta = 0$. From Fig.6, it is obvious to see that whenever $\gamma(t)$ (dotted line) takes negative values, E_a (solid line) begins to revive and increase monophonically, and the corresponding E_r begins to decrease monophonically; when $\gamma(t)$ takes positive values, E_a will begin to decrease monophonically, and E_r begins to increase monophonically. This can be easily understood: When $\gamma(t)$ is positive, the information flow is from atomic subsystem to reservoir subsystem, which means that E_a will decay, and E_r will increase; When $\gamma(t)$ is negative corresponding to the memory effect of the reservoir, the information flow is from the reservoir subsystem to atomic subsystem, so E_a will revive and E_r will decay.

4. Conclusions

In this paper, we have considered four atoms with initial entanglement interact with independent non-Markovian reservoirs. We have analyzed the decoherence dynamics for various initial states in Markovian ($\lambda = 10\gamma_0$), weak non-Markovian ($\lambda = 0.1\gamma_0$) and strong non-Markovian ($\lambda = 0.01\gamma_0$) regimes, with and without the detunings. We have found that the decoherence dynamics of the atomic and reservoir subsystems strongly depends on a parameter, which is decided by the detuning and the reservoir correlation time, and all the phenomenon can be explained by this parameter. The real part of this parameter determines the decay the entanglement and the imaginary part of it

determines the oscillations of the revival. We also have found that whenever $\gamma(t)$ takes negative values, E_a will begins to revive, and the corresponding E_r begins to decrease; when $\gamma(t)$ takes positive values, E_a will begin to decay, and E_r will begin to increase. We have also found that for this decoherence channel the cluster state is the most robust to decoherence comparing with Dicke, GHZ, and W states.

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References

[1] Ficek Z and Tanaś R 2006 Phys. Rev. A **74** 024304
[2] Dodd P J and Halliwell J J 2004 Phys. Rev. A **69** 052105
[3] Yu T and Eberly J H 2004 Phys. Rev. Lett. **93** 140404
[4] Sun Z, Wang X and Sun C-P 2007 Phys. Rev. A **75** 062312
[5] López C E, Romero G, Lastra F, Solano E and Retamal J C 2008 Phys. Rev. Lett. **101** 080503
[6] Al-Qasimi A and James D F V 2008 Phys. Rev. A **77** 012117
[7] Yu T and Eberly J H 2009 Science **323** 598
[8] Almeida M P, de Melo F, Hor-Meyll M, Salles A, Walborn S P, Souto Ribeiro P H and Davidovich L 2007 Science **316** 579
[9] Horodecki R, Horodecki P, Horodecki M and Horodecki K 2009 Rev. Mod. Phys.**81** 865
[10] Amico L, Fazio R, Osterloh A and Vedral V 2008 Rev. Mod. Phys. **80** 517
[11] Perez-Garcia D, Verstraete F, Wolf M M and Cirac J I 2007 Quantum Inf. Comput. **7** 401
[12] Gühne O, Bodoky F and Blaauboer M 2008 Phys. Rev. A **78** R060301
[13] Borras A, Majtey A P, Plastino A R, Casas M and Plastino A 2009 Phys. Rev. A **79** 022108
[14] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[15] Majtey A P, Borras A, Plastino A R, Casas M and Plastino A e-print arXiv:quant-ph/0904.1422v2.
[16] Breuer H-P and Petruccione F 2002 *The theory of Open Quantum systems* (Oxford: Oxford University Press)
[17] Lambropoulos P, Nikolopoulos G M, Nielsen T R and Bay S 2000 Rep. Prog. Phys. **63** 455
[18] Gardiner C W and Zoller P 1999 *Quantum noise* (Berlin: Springer-Verlag)
[19] Lai C W, Maletinsky P, Badolato A and Imamoglu A 2006 Phys. Rev. Lett.**96** 167403
[20] Shao J 2004 J. Chem. Phys. **120** 5053
[21] Aharonov D, Kitaev A and Preskill J 2006 Phys. Rev. Lett. **96** 050504
[22] Mancini S, Man'ko V I and Wiseman H M 2005 J. Phys. B **5** S177
[23] Ban M 2006 J. Phys. A **39** 1927
[24] Maniscalco S, Olivares S and Paris M G A 2007 Phys. Rev. A **75** 062119
[25] Liu K-L and Goan H-S 2007 Phys. Rev. A **76** 022312
[26] An J-H and Zhang W-M 2007 Phys. Rev. A **76** 042127
[27] Shiokawa K 2009 Phys. Rev. A **79** 012308
[28] Vasile R, Olivares S and Paris M G A, Maniscalco S 2009 Phys. Rev. A **80** 062324
[29] Breuer H-P 2004 Phys. Rev. A **70** 012106
[30] Piilo J, Maniscalco S, Häarkönen K and Suominen K A 2008 Phys. Rev. Lett. **100** 180402
[31] Breuer H-P and Piilo J 2009 Europhys. Lett. **85** 50004
[32] Mazzola L, Maniscalco S, Piilo J,Suominen K A and Garraway B M 2009 Phys. Rev. A **79** 042302
[33] Bellomo B, Franco R L and Compagno G 2007 Phys. Rev. Lett **99** 160502

[34] Zhang Y-J, Man Z-X and Xia Y-J 2009 Eur. Phys. J. D **55**173
[35] Zhang Y-J, Man Z-X, Zou X-B, Xia Y-J and Guo G-C, J. Phys. B **43** 045502
[36] Zhou J, Wu C-J, Zhu M-Y and Guo H 2009 J. Phys. B **42** 215505
[37] Hamadou-Ibrahim1 A, Plastino A R and Zander C, 2010 J. Phys. A **43** 055305
[38] Breuer H-P, Laine E M, and Piilo J 2009 Phys. Rev. Lett. **103** 210401
[39] Lu X-M, Wang X-G, and Sun C-P e-print arXiv:quant-ph/0912.0587v4
[40] Peres A 1996 Phys. Rev. Lett **77** 1413
[41] Horodecki M, Horodecki P and Horodecki R 1996 Phys. Lett. A **223** 1
[42] Wootters W K 1998 Phys. Rev. Lett. **80** 2245
[43] Vedral V and Plenio M B 1998 Phys. Rev. A **57** 1619
[44] Eisert J and Briegel H-J 2001 Phys. Rev. A **64** 022306
[45] Wei T-C and Goldbart P M 2003 Phys. Rev. A **68** 042307
[46] Meyer D A and Wallach N R 2002 J. Math. Phys. **43** 4273
[47] Hassan Ali Saif M and Joag P S 2008 Phys. Rev. A **77** 062334
[48] van Wonderen A J and Lendi K 2000 J. Stat. Phys. **100** 633
[49] Wolf M M, Eisert J, Cubitt T S and Cirac J I 2008 Phys. Rev. Lett. **101** 150402

CAPTIONS

Figure 1: In the Markovian regime ($\lambda = 10\gamma_0$, $\delta = 0$) E_a and E_r as functions of scaled time $\gamma_0 t$ for various initial states: (a) the atomic subsystem; (b) the reservoir subsystem.

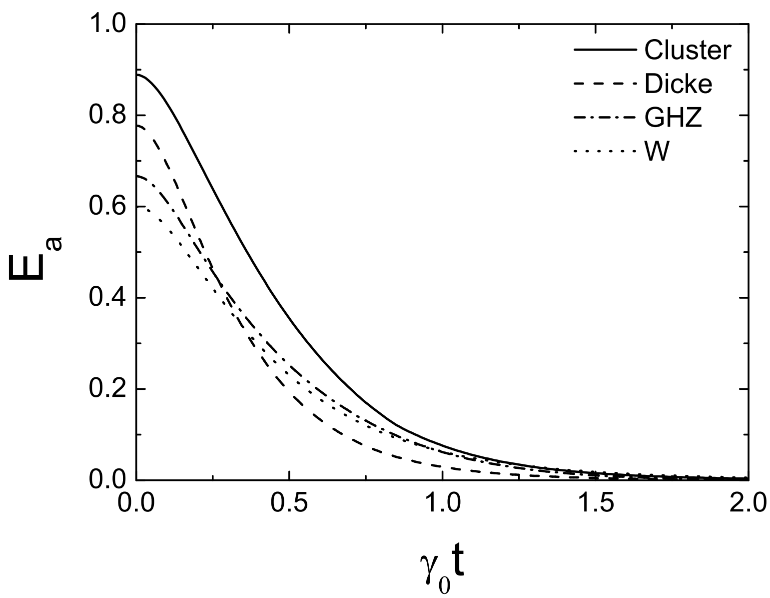
Figure 2: In the non-Markovian regime with relatively short reservoir correlation time ($\lambda = 0.1\gamma_0$, $\delta = 0$) E_a and E_r as functions of scaled time $\gamma_0 t$ for various initial states: (a) the atomic subsystem; (b) the reservoir subsystem.

Figure 3: In the non-Markovian regime with relatively long reservoir correlation time ($\lambda = 0.01\gamma_0$, $\delta = 0$) E_a and E_r as functions of scaled time $\gamma_0 t$ for various initial states: (a) the atomic subsystem; (b) the reservoir subsystem.

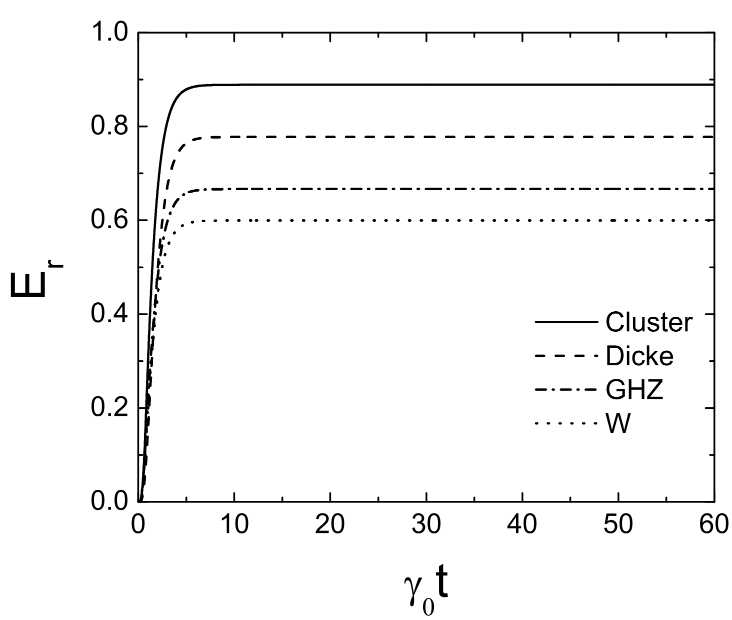
Figure 4: In the non-Markovian regime with relatively short reservoir correlation time with detuning ($\lambda = 0.1\gamma_0$, $\delta = 8\lambda$) E_a and E_r as functions of scaled time $\gamma_0 t$ for various initial states: (a) the atomic subsystem; (b) the reservoir subsystem.

Figure 5: In the non-Markovian regime with relatively long reservoir correlation time with detuning ($\lambda = 0.01\gamma_0$, $\delta = 8\lambda$) E_a and E_r as functions of scaled time $\gamma_0 t$ for various initial states: (a) the atomic subsystem; (b) the reservoir subsystem.

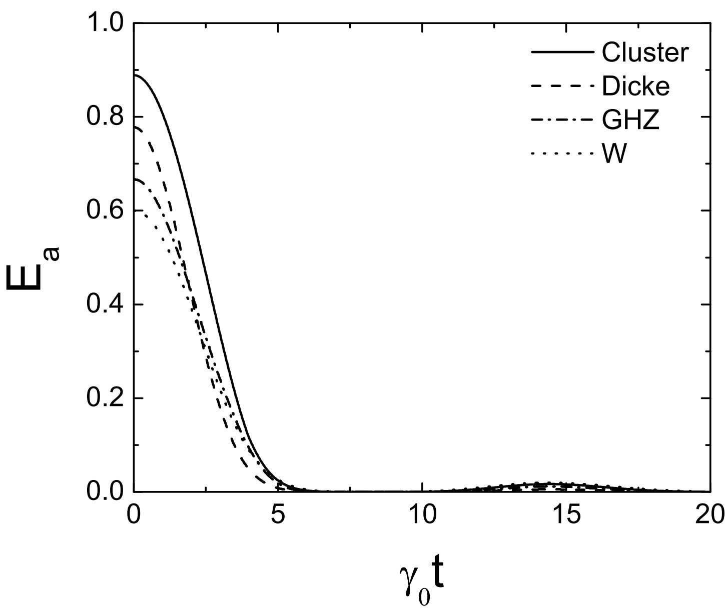
Figure 6: The atomic entanglement E_a, reservoir entanglement E_r and the decay rate $\gamma(t)$ for initial W state as functions of $\gamma_0 t$ ($\lambda = 0.01\gamma_0$ and $\delta = 0$): the atomic subsystem (solid line); the reservoir subsystem (dashed line); the decay rate (dotted line).



1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
E_a
$\gamma_0 t$
Cluster
Dicke
GHZ
W



1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
40
50
60
E_r
$\gamma_0 t$
Cluster
Dicke
GHZ
W



1.0
0.8
0.6
0.4
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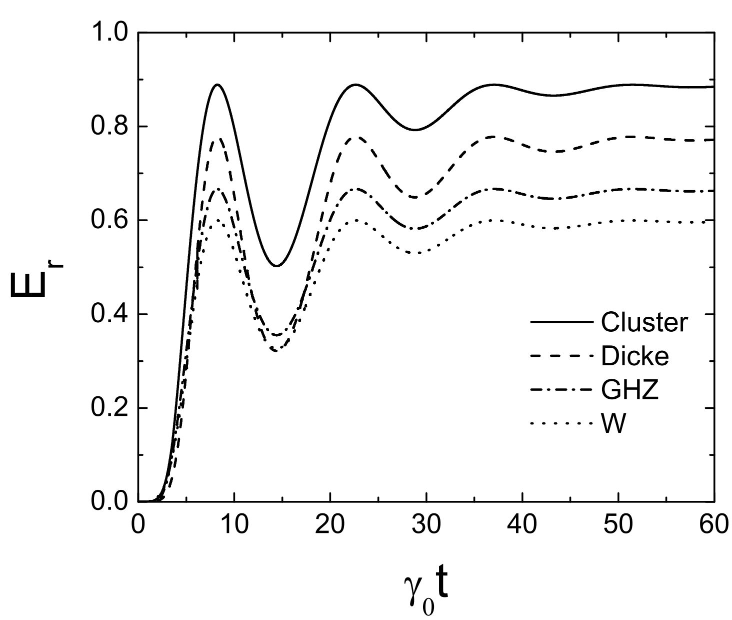
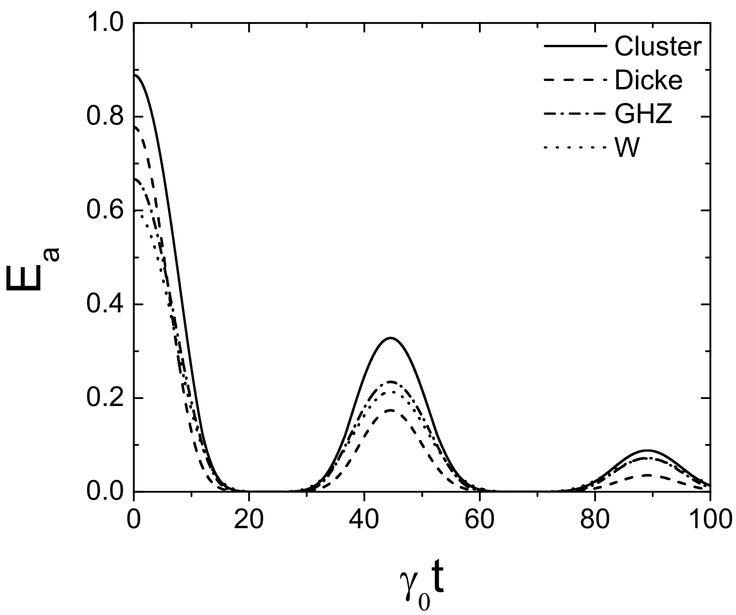
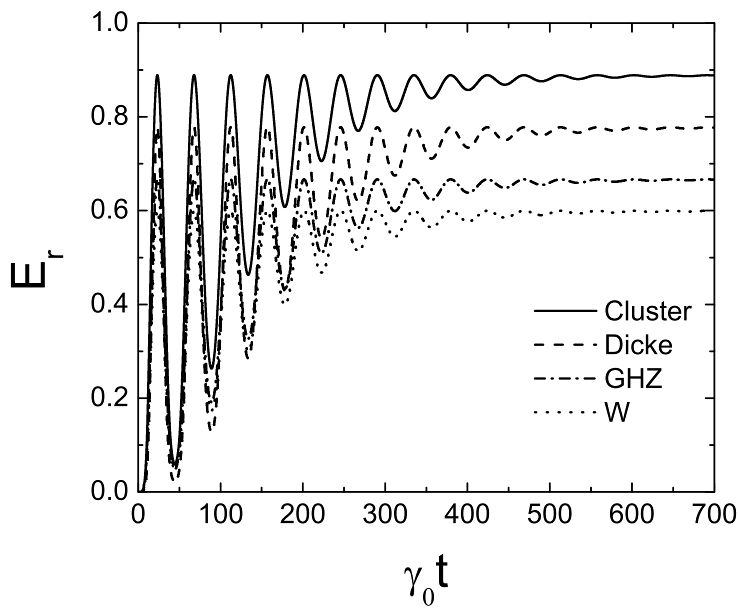
0.0
0
5
10
15
20
E_a
$\gamma_0 t$
Cluster
Dicke
GHZ
W



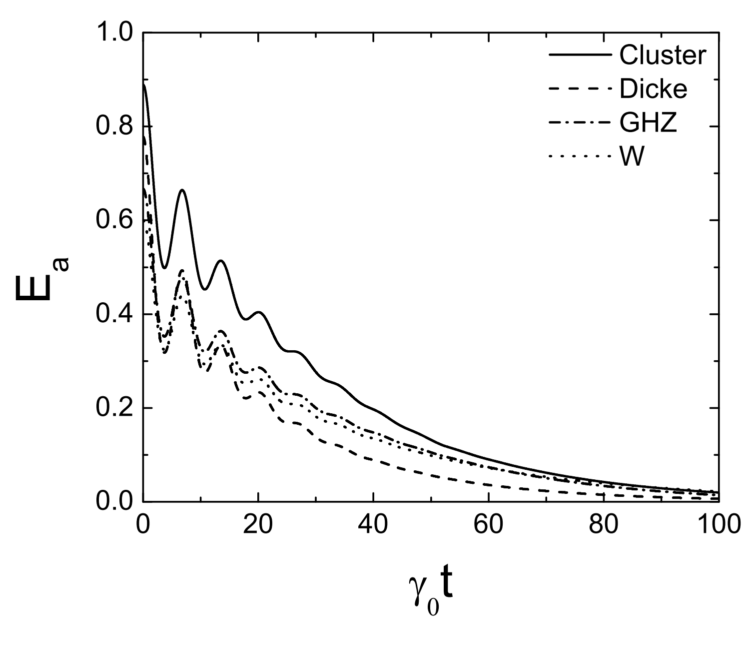
1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
40
50
60
E_r
$\gamma_0 t$
Cluster
Dicke
GHZ
W



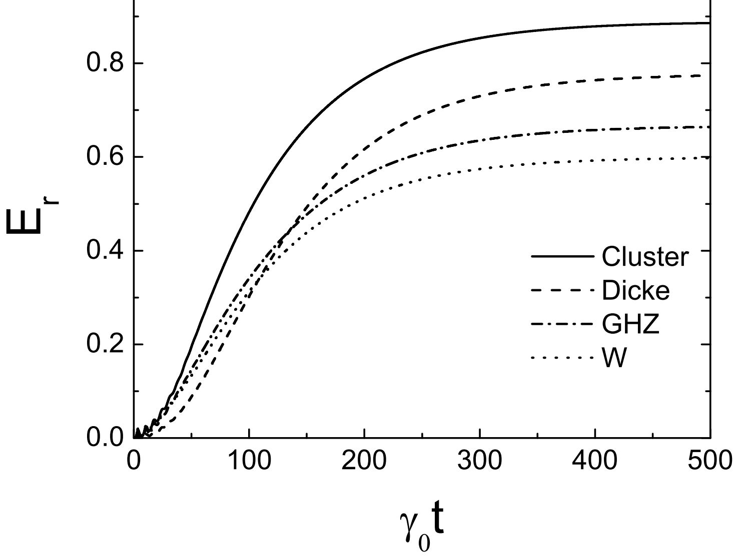
Cluster
Dicke
GHZ
W
E_a
$\gamma_0 t$
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100



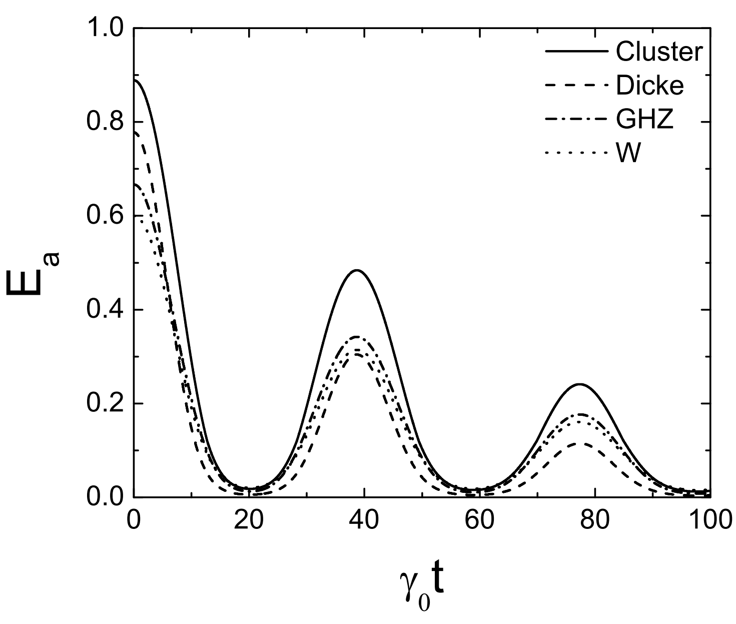
1.0
0.8
0.6
0.4
0.2
0.0
0
100
200
300
400
500
600
700
E_r
$\gamma_0 t$
Cluster
Dicke
GHZ
W



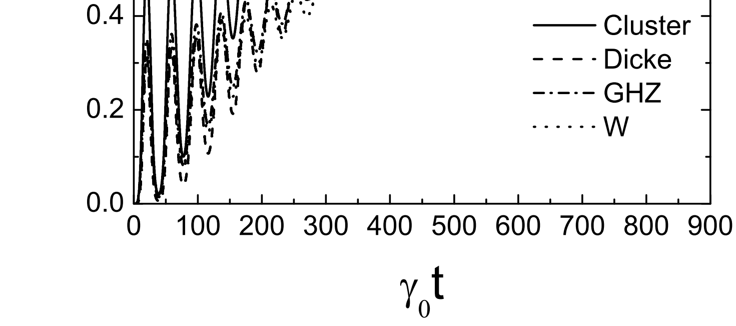
Cluster
Dicke
GHZ
W
E_a
$\gamma_0 t$
0.0
0.2
0.4
0.6
0.8
1.0
0
20
40
60
80
100



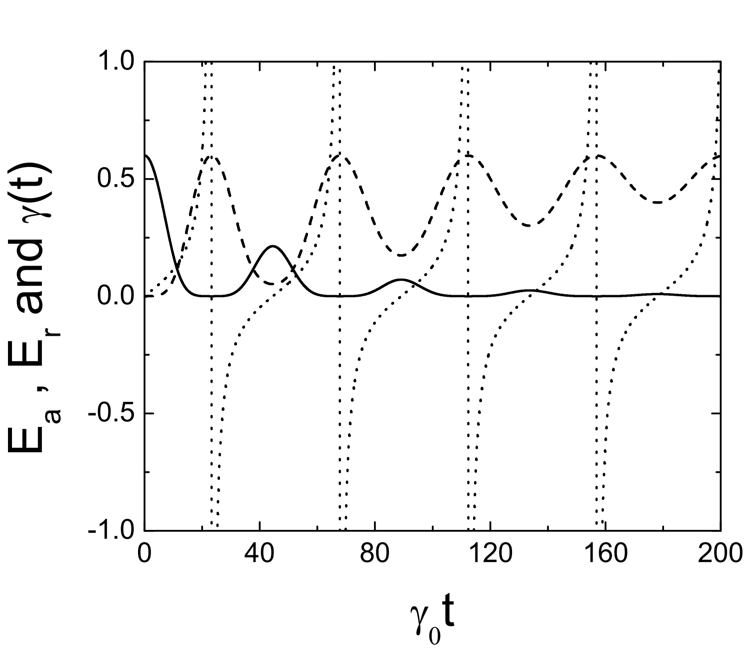
1.0
0.8
0.6
0.4
0.2
0.0
E_r
0
100
200
300
400
500
$\gamma_0 t$
Cluster
Dicke
GHZ
W



Cluster
Dicke
GHZ
W
E_a
$\gamma_0 t$
0.0
0.2
0.4
0.6
0.8
1.0
0
20
40
60
80
100



1.0
0.8
0.6
0.4
0.2
0.0
0
100
200
300
400
500
600
700
800
900
E_r
$\gamma_0 t$
Cluster
Dicke
GHZ
W



1.0
0.5
0.0
-0.5
-1.0
0
40
80
120
160
200
E_a , E_r and $\gamma(t)$
$\gamma_0 t$